Problems in laser physics

Sheet 3

Handed out on 9. 11. 17 for the Tutorial on 7. 12. 17

<u>Problem 7: Feedback condition and cw laser operation (4P)</u>

A 0.9% doped Yb^{3+}:YAG laser medium is inserted into a laser resonator consisting of a totally reflective mirror and a 80% reflectivity output coupler. Pump and laser modes are assumed to have a homogeneous intensity over a 2 mm diameter spot along the 40 mm-long laser rod. The intracavity loss was measured to $\Lambda = 2\%$ and is assumed to be located at the highly reflecting mirror.

(a) What single-pass gain G is necessary to obtain self-consistent operation? (1P)
(b) Derive the average populations of the two manifolds during laser operation and the pump absorption efficiency (2P).
(c) Calculate the laser threshold and the slope efficiency (1P).

<u>Problem 8: Caird plot (4P)</u>

The internal losses of a laser resonator and the intrinsic laser slope efficiency, i.e. the slope efficiency the same laser would have without losses, can be derived by a simple graphical method. Therefore, the inverse laser slope efficiencies with respect to the absorbed pump power $\frac{1}{\eta_{s,abs}}$ are determined for various output coupler reflectivities experimentally and are plotted versus the inverse output coupling $\frac{1}{T_{OC}}$. For low intracavity losses this plot corresponds to a staight line with a slope connected to the intracavity losses Λ. The intersection with the $\frac{1}{\eta_{s,abs}}$-axis, i.e. for $\frac{1}{T_{OC}} \to 0$, gives the intrinsic absorbed slope efficiency as $\frac{1}{\eta_{s,abs,0}}$.

(a) Rewrite and simplify the expression $\ln G$ as a sum in which one term contains $\ln G_0$, corresponding to the lossless resonator with $\Lambda = 0$ and derive the exact relation between $\frac{1}{\eta_{s,abs}}$ and $\frac{1}{T_{OC}}$ (1P).

(b) Use this result to derive the linear relation between $\frac{1}{\eta_{s,abs}}$ and $\frac{1}{T_{OC}}$ for $\Lambda \ll 1$, $T_{OC} \ll 1$ and $R_{HR} = 1$. What is $\frac{1}{\eta_{s,abs,0}}$ in this case (2P).
(c) In an Er^{3+}:YAG laser pumped at $\lambda_p = 1530$ nm and emitting at $\lambda_s = 1645$ nm, OC reflectivities of 60%, 70%, 95%, and 98% resulted in absorbed slope efficiencies of 0.4, 0.334, 0.129 and 0.06, respectively. Determine the intrinsic slope efficiency and the internal cavity losses. (1P)

Problem 9: Spontaneous emission and vacuum noise (4P)

In the lecture it was shown that the spontaneous emission into the resonator mode can be treated as a stimulated emission when we assume that the mode itself shows a vacuum noise spectral photon density of

$$\tilde{\Phi}_0 = \frac{\Delta\Omega_s}{4\pi}\frac{8\pi n^2}{\lambda^4} \,, \tag{1}$$

with $\Delta\Omega_s$ being the solid-angle of the corresponding mode. The change in the spectral photon density of the cavity due to spontaneous emission thus is given by

$$\left.\frac{\partial\langle\tilde{\Phi}\rangle}{\partial t}\right|_{spont} = c\sigma_e(\lambda)\langle N_2\rangle\tilde{\Phi}_0 \,. \tag{2}$$

The total change of the photon density of the cavity due to spontaneous emission then corresponds to the integral

$$\left.\frac{\partial\langle\Phi\rangle}{\partial t}\right|_{spont} = \int \left.\frac{\partial\langle\tilde{\Phi}\rangle}{\partial t}\right|_{spont} d\lambda \,. \tag{3}$$

Another simple way of introducing the spontaneous emission into the photon density rate equation is by counting the number of spontaneously emitted photons into the mode solid angle, resulting in

$$\left.\frac{\partial\langle\Phi\rangle}{\partial t}\right|_{spont} = \frac{\Delta\Omega_s}{4\pi}\frac{\langle N_2\rangle}{\tau_{21}} \tag{4}$$

(a) Show that both ways are equivalent by deducing Eq. (4) from Eq. (3). (2P)

A solid angle $\Delta\Omega$ of a cylindrically symmetric cone is related to its opening half angle θ by $\Delta\Omega = 2\pi(1 - \cos\theta)$. This allows to find a relation for the solid angle of the fundamental mode, the Gaussian beam, which shows a divergence of $\theta = \frac{\lambda}{\pi w_0 n}$ (half angle) inside a medium of refractive index n when being focused to a waist w_0, i.e. to an effective beam area of $A = \pi w_0^2$.
(b) Using this relation, show that the vacuum noise of the mode corresponds to a noise power of a single photon per polarization. Hint: $\theta \ll 1$. (2P)